

# Nested Dynamic Condition Response Graphs

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**Abstract.** We present an extension of the recently introduced declarative process model *Dynamic Condition Response Graphs* (DCR Graphs) to allow *nested sub-graphs* and a new *milestone relation* between events. The extension was developed during a case study carried out jointly with our industrial partner Exformatics, a danish provider of case and workflow management systems. We formalize the semantics by giving first a map from Nested to (flat) DCR Graphs with milestones, and then extending the previously given mapping from DCR Graphs to Büchi-automata to include the milestone relation.

## 1 Introduction

Declarative process models have been suggested by several research groups [1–5, 15, 16, 18, 19] as a good approach to describe case management and other non-rigid business and workflow processes where it is generally allowed to redo or skip activities, and even dynamically adapt the set of activities and constraints. The rationale is that if a strict sequencing is the exception, then the implicit specification of control flow in declarative models is more appropriate than notations based on explicit control flows such as the (typical use of) Business Process Model and Notation (BPMN) 2.0 [13].

A drawback of the declarative approaches in general, however, is that the implicit definition of the state and control flow makes it more complex to perceive the state and execute the process. To find out what are the next possible activities it is necessary to evaluate a set of constraints defined relatively to the history of the execution.

This motivates finding an expressive declarative process language that allows for a simple run-time scheduling which is easily visualized for the case worker. As a candidate for such a language we recently introduced in [7, 11] a declarative process model called Dynamic Condition Response Graphs (DCR Graphs). The model is a generalization of the classic event structure model for concurrency [20] and is inspired by the Process Matrix model [10, 12] developed by one of our industrial partners Resultmaker, a Danish provider of workflow and case-management systems.

The core DCR Graphs model consists of a set of events and four binary relations between the events: The *dynamic inclusion* and *dynamic exclusion* relations, and the *condition* and *response* relations. The dynamic inclusion and exclusion relations generalize the usual symmetric conflict relation of event structures by splitting it in two

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the dates. But it also enables that DA can propose other dates, represented by the event **Propose Dates-DA**. Now, this event triggers as a response and includes the event **Accept LO**, representing LO accepting the dates. Again, LO may do this, or again propose dates. The proposal of dates may continue forever, and as long as no one accepts there will be a pending response on at least one of the accept events. As soon as one of the accept events happen, they will both be excluded, and there will be none of the included events in the sub-graph having pending responses. This corresponds to the accepting condition for finite runs of DCR graphs [7], and thus intuitively reflects that the sub-graph is in a completed state. Now, we want to express that the event **Hold meeting** can only be executed when this is the case. To do this, we introduced a new core relation between events called the *milestone* relation. If an event  $A$  is a milestone for an event  $B$ , written  $A \rightarrow \diamond B$ , then  $B$  can not happen if  $A$  is included and required to be executed again (i.e. as a response). The new milestone relation allow us to define nesting as simply a tree structure on events that can be flattened to (flat) DCR Graphs by keeping all atomic events (i.e. events with no sub-events) and letting them inherit the relations defined for their super-events. In particular, the flattening does not introduce new events (in fact it removes all super events) and at most introduce an order of  $n^2$  new relations. Thus, we need not define a new operational semantics for nested DCR Graphs, instead we can make the much simpler extension of the semantics for (flat) DCR Graphs to consider the new milestone relation. It is worth noting that while the milestone relation makes it very direct to express completion of subgraphs, we conjecture that it does not add expressiveness to DCR Graphs.

*Related work:* Our approach is closely related to the work on ConDec [18, 19]. The crucial difference is that we allow nesting and a few core constraints making it possible to describe the state of a process as a simple marking. ConDec does not address nesting (nor dynamic inclusion/exclusion), but allows one to specify any relation expressible within Linear-time Temporal Logic (LTL). This offers much flexibility with respect to specifying execution constraints. In particular the condition and response relations are standard verification patterns and also considered in [18, 19] (the condition relation is called precedence), and we have used the same graphical notation. However, the execution of a process expressed as LTL (which typically involves a translation to a Büchi-automaton) is more complex and the run-time state is difficult to relate to the original ConDec specification. Moreover, we conjecture that DCR Graphs are as expressive as Büchi-automata, and thus more expressive than LTL. Finally, Nested DCR Graphs relates to the independent (so far unpublished) work on the declarative *Guard-Stage-Milestone* model by Hull, presented in invited talks at WS-FM 2010 and CASCON 2010.

*Structure of paper:* In Sec. 2 we define Nested DCR Graphs formally, motivated by the case study, and define the mapping to flat DCR Graphs with milestones. In Sec. 3 we then define the lts semantics and the mapping from flat DCR Graphs with milestones to Büchi-automata. The two maps together define the semantics of Nested DCR Graphs. Due to space limitations we refer to [8] and the full version [9] for a detailed description of the case study and tool support. We conclude in Sec. 4 and give pointers to future work.

## 2 Nested DCR Graphs and Milestones

We now give the formal definition of the Nested DCR Graph model described informally above, which extends the model in our previous work [7] with nesting and the new milestone relation  $\rightarrow\circ$  between events.

**Definition 1.** A Nested Distributed dynamic condition response graph with milestones is a tuple  $(E, \triangleright, M, \rightarrow\bullet, \bullet\rightarrow, \rightarrow\circ, \pm, \text{Act}, l, R, P, \text{as})$ , where

- (i)  $E$  is the set of events
- (ii)  $\triangleright : E \rightarrow E$  is a partial function mapping an event to its super-event (if defined), and we also write  $e \triangleright e'$  if  $e' = \triangleright^k(e)$  for  $0 < k$ , referred to as the nesting relation
- (iii)  $M = (E, R, I) \subseteq \text{atoms}(E) \times \text{atoms}(E) \times \text{atoms}(E)$  is the marking, containing sets of currently executed events ( $E$ ), currently pending responses ( $R$ ), and currently included events ( $I$ ).
- (iv)  $\rightarrow\bullet \subseteq E \times E$  is the condition relation
- (v)  $\bullet\rightarrow \subseteq E \times E$  is the response relation
- (vi)  $\rightarrow\circ \subseteq E \times E$  is the milestone relation
- (vii)  $\pm : E \times E \rightarrow \{+, \%\}$  is a partial function defining the dynamic inclusion and exclusion relations by  $e \rightarrow+ e'$  if  $\pm(e, e') = +$  and  $e \rightarrow\% e'$  if  $\pm(e, e') = \%$
- (viii)  $\text{Act}$  is the set of actions
- (ix)  $l : E \rightarrow \text{Act}$  is a labeling function mapping events to actions.
- (x)  $R$  is a set of roles,
- (xi)  $P$  is a set of principals (e.g. persons or processors) and
- (xii)  $\text{as} \subseteq (P \cup \text{Act}) \times R$  is the role assignment relation to principals and actions.

where  $\text{atoms}(E) = \{e \mid \forall e' \in E. \triangleright(e') \neq e\}$  is the set of atomic events.

We require that the nesting relation  $\triangleright \subset E \times E$  is acyclic and that there are no infinite sequence of events  $e_1 \triangleright e_2 \triangleright \dots$ . We will write  $e \triangleright e'$  if  $e \triangleright e'$  or  $e = e'$ , and  $e \leq e'$  if  $e' \triangleright e$  or  $e = e'$ . We require that the nesting relation is consistent with respect to dynamic inclusion/exclusion in the following sense: If  $e \triangleright e'$  or  $e' \triangleright e$  then  $\pm(e, e'') = +$  implies  $\pm(e', e'') \neq \%$  and  $\pm(e, e'') = \%$  implies  $\pm(e', e'') \neq +$ .

The new elements are the nesting relation  $\triangleright \subset E \times E$  and the milestone relation  $\rightarrow\circ \subseteq E \times E$ . The consistency between the nesting relation and the dynamic inclusion/exclusion is to ensure that when we map a nested DCR Graph to the corresponding flat DCR Graph as defined in Def. 2 below, no atomic event both includes and excludes another event. That is, if an event  $e$  includes (excludes) another event  $e''$ , then any of its super or sub events  $e'$  can not exclude (include) the event  $e''$ .

The new elements conservatively extend the DCR Graphs defined in [7] in the sense that given a Nested dynamic condition response graph as defined in Def. 1, the tuple  $(\text{atoms}(E), M, \rightarrow\bullet, \bullet\rightarrow, \pm, \text{Act}, l, R, P, \text{as})$  is a (Distributed) dynamic condition response graph as defined in [7]. In particular, the semantics will be identical if both the  $\triangleright$  map and the milestone relation are empty.

A nested distributed dynamic condition response graph can be mapped to a flat distributed dynamic condition response graph with at most the same number of events and a quadratic growth of the relations. Essentially, all relations are extended to sub

events, and then only the atomic events are preserved. The labelling function is extended by labelling an atomic event  $e$  by the sequence of labels labelling the chain of super events starting by the event itself:  $e \triangleright e_1 \dots \triangleright e_k \not\triangleright$ . The role assignment is extended to sequences of actions by taking the union of roles assigned to the actions.

**Definition 2.** For a Nested DCR Graph  $G = (E, \triangleright, M, \rightarrow\bullet, \bullet\rightarrow, \rightarrow\circ, \pm, \text{Act}, l, R, P, \text{as})$  define the underlying flat DCR Graph as

$$G^b = (\text{atoms}(E), M, \rightarrow\bullet^b, \bullet\rightarrow^b, \rightarrow\circ^b, \pm^b, \text{Act}^+, l^b, R, P, \text{as}^b),$$

where

- $rel^b = \triangleright rel \triangleleft$  for  $rel \in \{\rightarrow\bullet, \bullet\rightarrow, \rightarrow\circ\}$ ,
- $\pm(e, e')^b = \pm(e_s, e'_s)$  if  $\pm(e_s, e'_s)$  is defined,  $e \triangleright e_s$  and  $e' \triangleleft e'_s$ ,
- $l^b(e_0) = a_0.a_1.a_2 \dots a_k$  and  $\text{as}^b(a_0.a_1.a_2 \dots a_k) = \bigcup_{0 \leq i \leq k} \text{as}(a_i)$ , if  $e_0 \triangleright e_1 \triangleright e_2 \dots \triangleright e_k \not\triangleright$  and  $l(e_i) = a_i$ , for  $0 \leq i \leq k$ , and
- $\text{as}^b(p) = p$  for  $p \in P$ .

### 3 Semantics

Below we define the semantics of DCR Graphs with milestones by giving a labelled transition semantics and a mapping to Büchi-automata.

**Notation:** For a set  $A$  we write  $\mathcal{P}(A)$  for the power set of  $A$ . For a binary relation  $\rightarrow \subseteq A \times A$  and a subset  $\xi \subseteq A$  of  $A$  we write  $\rightarrow \xi$  and  $\xi \rightarrow$  for the set  $\{a \in A \mid (\exists a' \in \xi \mid a \rightarrow a')\}$  and the set  $\{a \in A \mid (\exists a' \in \xi \mid a' \rightarrow a)\}$  respectively.

**Definition 3.** For a dynamic condition response graph with milestones  $G = (E, M, \rightarrow\bullet, \bullet\rightarrow, \rightarrow\circ, \pm, l, \text{Act}, R, P, \text{as})$ , we define the corresponding labelled transition systems  $T(G)$  to be the tuple  $(S, M, \rightarrow \subseteq S \times \text{Act} \times S)$  where  $S = \mathcal{P}(E) \times \mathcal{P}(E) \times \mathcal{P}(E)$  is the set of markings of  $G$  and  $M = (R, I, E) \in S$  is the initial marking,  $\rightarrow \subseteq S \times E \times (P \times \text{Act} \times R) \times S$  is the transition relation given by  $M' \xrightarrow{(e, (p, a, r))} M''$  where

- (i)  $M' = (E', R', I')$  is the marking before transition
- (ii)  $M'' = (E' \cup \{e\}, R'', I'')$  is the marking after transition
- (iii)  $e \in I$ ,  $l(e) = a$ ,  $p \text{ as } r$ , and  $a \text{ as } r$ ,
- (iv)  $\rightarrow\bullet e \cap I' \subseteq E'$ ,
- (v)  $\rightarrow\circ e \cap I' \cap R' = \emptyset$ ,
- (vi)  $I'' = (I' \cup e \rightarrow +) \setminus e \rightarrow \%$ ,
- (vii)  $R'' = (R' \setminus \{e\}) \cup e \bullet \rightarrow$ ,
- (viii)  $E'' = E' \cup \{e\}$

We define a run  $(e_0, (p_0, a_0, r_0)), (e_1, (p_1, a_1, r_1)), \dots$  of the transition system to be a sequence of labels of a sequence of transitions  $M_i \xrightarrow{(e_i, (p_i, a_i, r_i))} M_{i+1}$  starting from the initial marking. We define a run to be accepting if  $\forall i \geq 0, e \in R_i, \exists j \geq i. (e = e_j \vee e \notin I_j)$ . In words, a run is accepting if no response event is included and pending forever, i.e. it must either happen at some later state or become excluded.

Condition (iii) in the above definition expresses that, only events  $e$  that are currently included, can be executed, and to give the label  $(p, a, r)$  the label of the event must be  $a, p$  must be assigned to the role  $r$ , which must be assigned to  $a$ . Condition (iv) requires that all condition events to  $e$  which are currently included should have been executed previously. Condition (v) states that the currently included events which are milestones to event  $e$  must not be in the set of pending responses ( $R'$ ). Conditions (vi), (vii) and (viii) are the updates to the sets of included events, pending responses and executed events respectively. Note that an event  $e'$  can not be both included and excluded by the same event  $e$ , but an event may trigger itself as a response.

If one considers only finite runs then the acceptance condition degenerates to requiring that no pending response is included at the end of the run. If infinite runs are also of interest (as e.g. for reactive systems and LTL) the acceptance condition can be captured by a mapping to a Büchi-automaton with  $\tau$ -event defined as follows.

**Definition 4.** A Büchi-automaton with  $\tau$ -event is a tuple  $(S, s, Ev_\tau, \rightarrow \subseteq S \times Ev_\tau \times S, F)$  where  $S$  is the set of states,  $s \in S$  is the initial state,  $Ev_\tau$  is the set of events containing the special event  $\tau$ ,  $\rightarrow \subseteq S \times Ev_\tau \times S$  is the transition relation, and  $F$  is the set of accepting states. A (finite or infinite) run is a sequence of labels not containing the  $\tau$  event that can be obtained by removing all  $\tau$  events from a sequence of labels of transitions starting from the initial state. The run is accepting if the sequence of transitions passes through an accepting state infinitely often.

Since we at any given time may have several pending responses we must make sure in the mapping to Büchi-automata that all of them are eventually executed or excluded. To do this we assume any fixed order of the finite set of events  $E$  of the given dynamic condition response graph. For an event  $e \in E$  we write  $rank(e)$  for its rank in that order and for a subset of events  $E' \subseteq E$  we write  $min(E')$  for the event in  $E'$  with the minimal rank.

**Definition 5.** For a finite distributed dynamic condition response graph  $G = (E, M, \rightarrow \bullet, \bullet \rightarrow, \rightarrow \circ, \pm, Act, l, R, P, as)$  where  $E = \{e_1, \dots, e_n\}$ , marking  $M = (E, R, I)$  and  $rank(e_i) = i$ , we define the corresponding Büchi-automaton with  $\tau$ -event to be the tuple  $B(G) = (S, s, \rightarrow \subseteq S \times Ev_\tau \times S, F)$  where

- $S = \mathcal{P}(E) \times \mathcal{P}(E) \times \mathcal{P}(E) \times \{1, \dots, n\} \times \{0, 1\}$  is the set of states,
- $Ev_\tau = (E \times (P \times Act \times R)) \cup \{\tau\}$  is the set of events,
- $s = (M, 1, 1)$  if  $I \cap R = \emptyset$ , and  $s = (M, 1, 0)$  otherwise
- $F = \mathcal{P}(E) \times \mathcal{P}(E) \times \mathcal{P}(E) \times \{1, \dots, n\} \times \{1\}$  is the set of accepting states and
- $\rightarrow \subseteq S \times Ev_\tau \times S$  is the transition relation given by  $(M', i, j) \xrightarrow{\tau} (M', i, j')$  where

- (a)  $M' = (E', R', I')$  is the marking
- (b)  $j' = 1$  if  $I' \cap R' = \emptyset$  otherwise  $j' = 0$ .

and  $(M', i, j) \xrightarrow{(e, (p, a, r))} (M'', i', j')$  where

- (i)  $M' = (E', R', I') \xrightarrow{(e, (p, a, r))} (E'', R'', I'') = M''$  is a transition of  $T(D)$ .

- (ii) For  $M = \{e \in I' \cap R' \mid \text{rank}(e) > i\}$  let  $j' = 1$  if
  - (a)  $I'' \cap R'' = \emptyset$  or
  - (b)  $\min(M) \in (I' \cap R' \setminus (I'' \cap R'')) \cup \{e\}$  or
  - (c)  $M = \emptyset$  and  $\min(I' \cap R') \in (I' \cap R' \setminus (I'' \cap R'')) \cup \{e\}$   
otherwise  $j' = 0$ .
- (iii)  $i' = \text{rank}(\min(M))$  if  $\min(M) \in (I' \cap R' \setminus (I'' \cap R'')) \cup \{e\}$  or else
- (iv)  $i' = \text{rank}(\min(I' \cap R'))$  if  $M = \emptyset$  and  $\min(I' \cap R') \in (I' \cap R' \setminus (I'' \cap R'')) \cup \{e\}$   
or else
- (v)  $i' = i$  otherwise.

We prove that the mapping from the labelled transition semantics to Büchi-automata is sound and complete in the full version of the paper [9].

The formal semantics of DCR graphs mapped to Büchi-automata enabled us to perform model checking and formal verification of processes specified in DCR graphs. The prototype implementation allows us to perform verification of both safety and liveness properties using the SPIN [17] model checker and only verification of safety properties using the ZING [14] model checker. The prototype has also been extended to support runtime verification, for monitoring of properties specified using Property Patterns [6].

## 4 Conclusion and Future Work

We have given a conservative extension of the declarative process model Distributed DCR Graphs [7] to allow for nested sub-graphs motivated and guided by a case study carried out jointly with our industrial partner. A detailed description of the case study and tool support for DCR Graphs can be found in [8]. The main technical challenge was to formalize the execution and in particular *completion* of sub-graphs. We do this by introducing a new *milestone* relation  $A \rightarrow \diamond B$ , which blocks the event  $B$  as long as there are events in  $A$  required to be executed (i.e. required responses). We believe this is the right notion of completeness of nested sub-graphs. First of all, it coincides with the definition of acceptance of finite runs in DCR Graphs [7] recalled in Sec. 3 above. Secondly, its formalization is a simple extension of the labelled transition semantics given in [7, 11] since it is a condition on the set of pending responses already included in the states. Finally, it allows for a nested sub-graph to alternate between being completed and not completed, as is often the case in ad hoc case management. This is not possible in the related ad-hoc sub-process activity in BPMN 2.0. Future work within the Trust-Care and CosmoBiz projects, which are the context of the work, includes exploring the expressiveness of DCR Graphs, extending the theory and tools for analysis, verification and model-driven engineering, extending the model to be able to express other relevant features such as multi-instance sub-graphs, time, exceptions, data, types and run-time adaptation, i.e. dynamic changes of the model.

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